Lecture 7

INDUCTORS

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INDUCTORS

- Faraday's Law states that: the magnitude of the induced voltage in a circuit is proportional to the rate of change of the flux passing through the circuit.
- Lenz's Law: the polarity of the induced voltage is such as to oppose the cause producing it.
- Counter emf or back voltage: the induced voltage that tries to opposes changes in current. This voltage does not stop the current from changing. It only stops sudden changes .

INDUCTORS

- The induced voltage depends on the coil core. Ferromagnetic cores confine the flux entirely. They are called iron-core coils. Nonmagnetic cores have same permeability as air and do not confine flux. They are called air core coils.
- For iron-core, the induced emf is given by:

$$e = N\frac{d\phi}{dt}$$

Where Flux φ lines are passing through N turns.

INDUCTORS

- For air core inductors, the induced emf is proportional to the rate of current change:

$$e = L\frac{dI}{dt}$$

Where L is the self inductance.

- Inductances in series will have a total inductance of

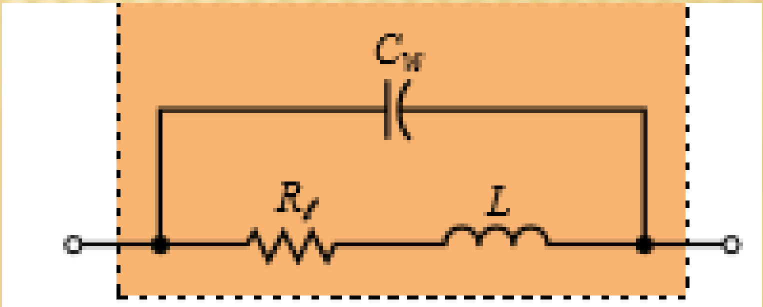
$$L_{tot} = L_1 + L_2 + L_3 + \ldots\ldots$$

- Inductance in parallel will have a total inductance of

$$\frac{1}{L_{tot}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots\ldots$$

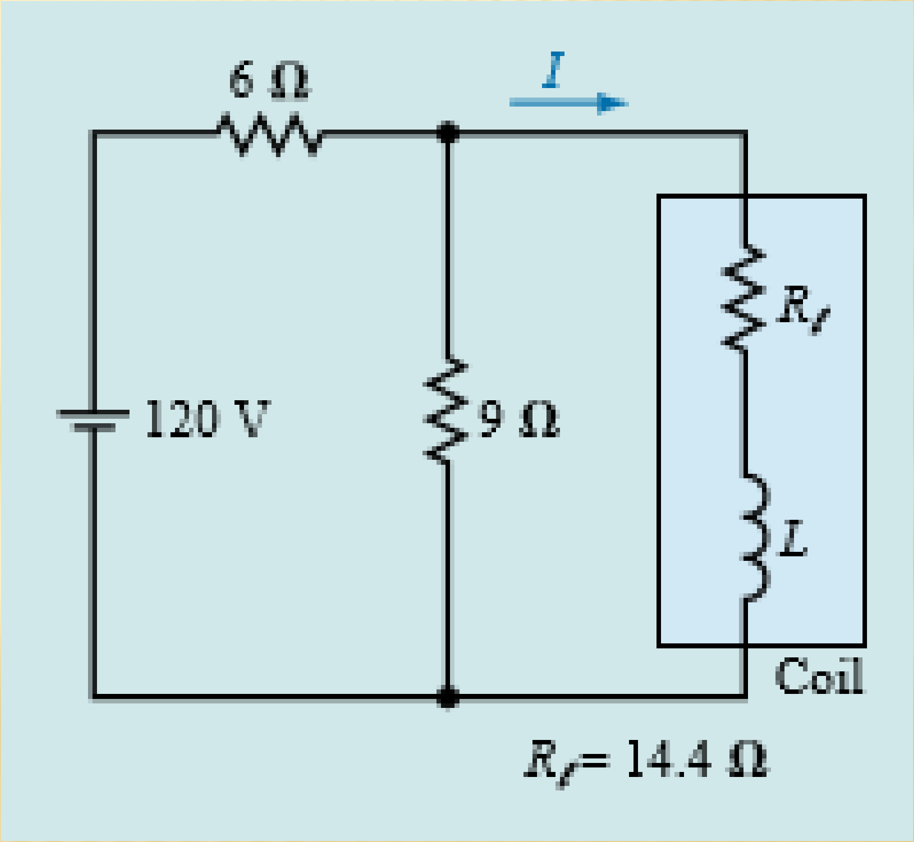
STRAY CAPACITANCE AND INDUCTANCE

- Coil resistance: since inductors are made up of imperfect conductors, they also resistance. This is usually graphed as a resistance in series.
- Stray capacitance for an inductor coil: it is produced because the inductor turns are separated from each other by insulation.
- Stray inductance: it is due to magnetic effects of electric current. All current carrying have inductance.



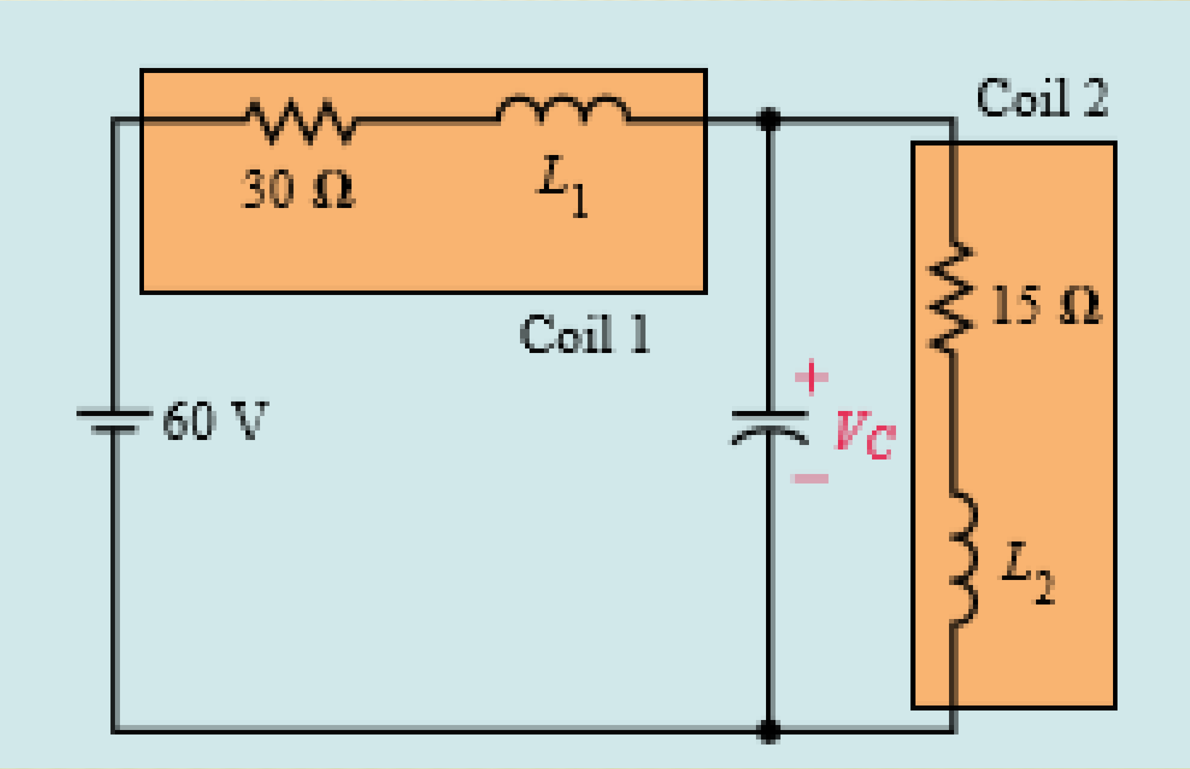
PROBLEM

- The coil resistance is 14.4. what is the steady state current?



PROBLEM

- The resistance of coil 1 is 30 and that of coil 2 is 15. Find the voltage across the capacitor assuming steady state dc?



ENERGY STORED

- Energy stored by an inductance

$$P = IV \Rightarrow W = \int P dt$$

$$W = \int IV dt = \int IL \frac{dI}{dt} dt = \int IL dI = \frac{1}{2} LI^2$$

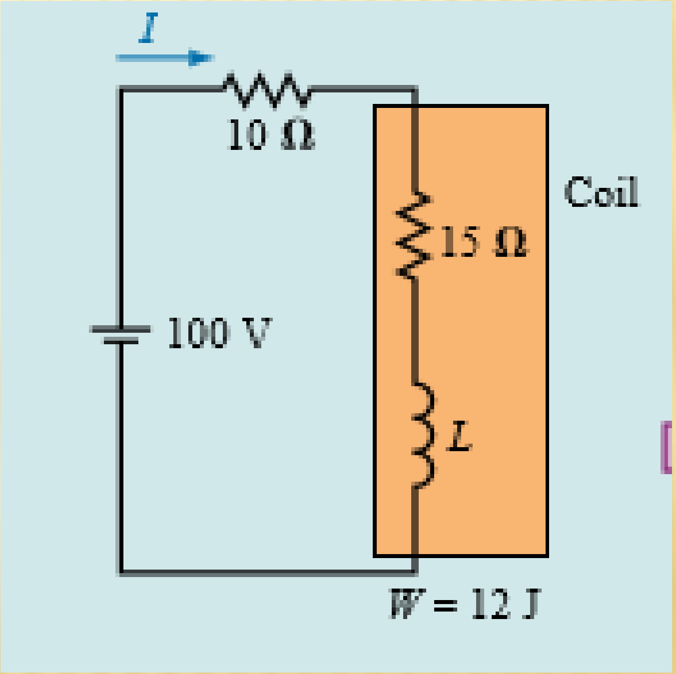
- Energy stored by a capacitance

$$P = IV \Rightarrow W = \int P dt$$

$$W = \int IV dt = \int VC \frac{dV}{dt} dt = \int CV dV = \frac{1}{2} CV^2$$

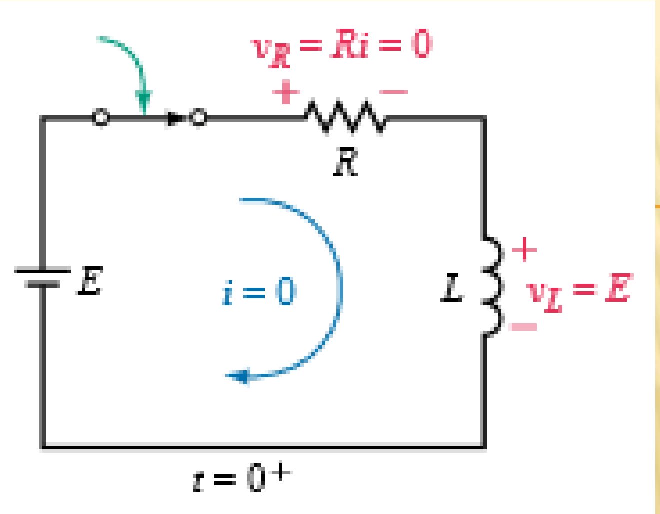
PROBLEM

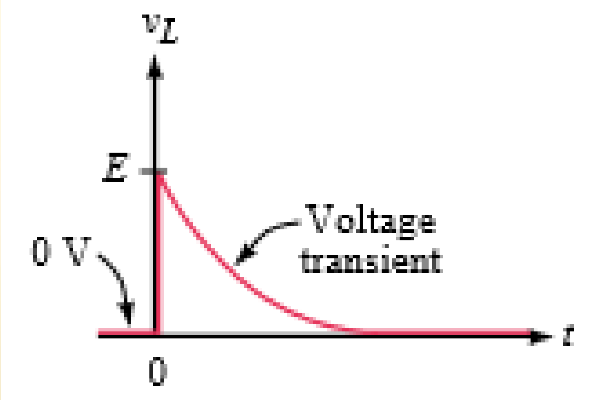
- The coil has a resistance of 15. when the current reaches its steady state value, the energy stored is 12 J. what is L?

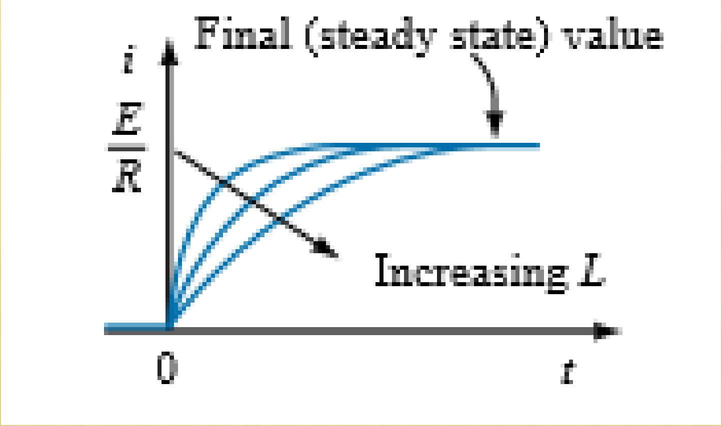


INDUCTANCE ENERGIZING

- When the switch is closed, induced back emf is produced. The voltage will decay to zero as the current reaches steady state.
- For the current, it starts building up from zero.







INDUCTOR ENERGIZING

$$V_L + V_R = E \Rightarrow V_L + IR = E$$

$$V_L = L\frac{dI}{dt} \Rightarrow L\frac{dI}{dt} + IR = E$$

$$\frac{dI}{dt} = -\frac{R}{L}(I - \frac{E}{R})$$

$$\frac{dI}{I - E/R} = -\frac{Rdt}{L}$$

$$\int_0^I \frac{dI}{I - E/R} = -\int_0^t \frac{Rdt}{L}$$

$$\ln(I - E/R) - \ln(-E/R) = -\frac{Rt}{L}$$

$$\ln\left(\frac{IR - E}{-E}\right) = -\frac{Rt}{L}$$

$$-\frac{IR}{E} + 1 = e^{-\frac{Rt}{L}}$$

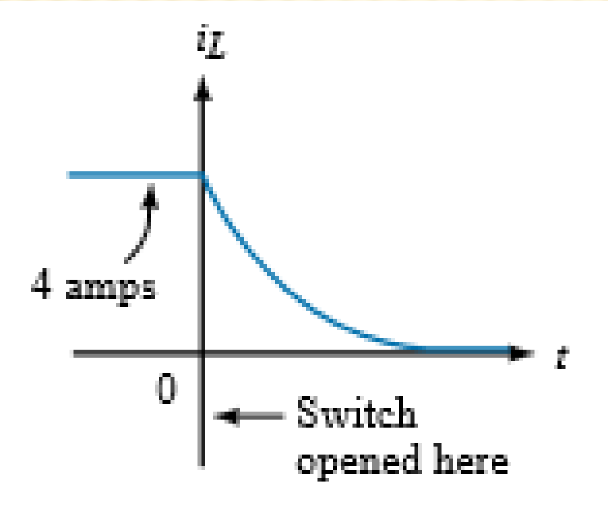
$$I = \frac{E}{R}(1 - e^{-Rt/L})$$

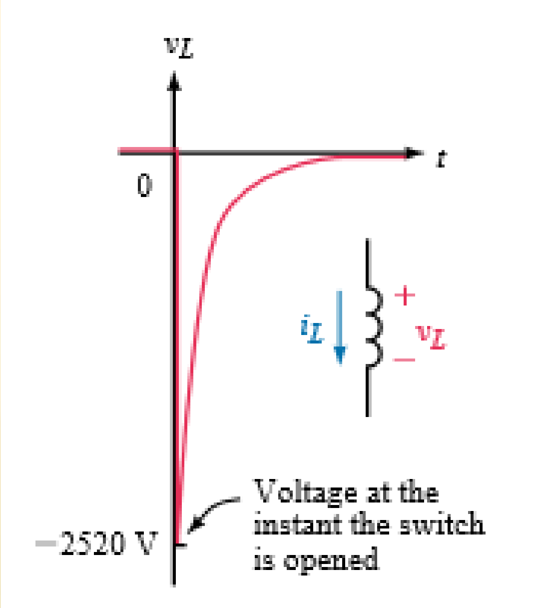
$$V_R = IR = E(1 - e^{-Rt/L})$$

$$V_L = E - V_R = Ee^{-Rt/L}$$

DE ENERGIZING CASE

- In this case, the current going through the inductance has an original value so the induced emf will have an opposite sign to before. It will gradually decay to zero again.
- The current, on the other hand, will decay gradually to zero.





INDUCTOR DE-ENERGIZING

$$V_L + V_R = 0 \Rightarrow V_L + IR = 0$$

$$V_L = L\frac{dI}{dt} \Rightarrow L\frac{dI}{dt} + IR = 0$$

$$\frac{dI}{dt} = -\frac{R}{L}I$$

$$\frac{dI}{I} = -\frac{Rdt}{L}$$

$$\int_{I_0}^{I}\frac{dI}{I} = -\int_0^t \frac{Rdt}{L}$$

$$\ln(I) - \ln(I_0) = -\frac{Rt}{L}$$

$$\ln\left(\frac{I}{I_0}\right) = -\frac{Rt}{L}$$

$$\frac{I}{I_0} = e^{-\frac{Rt}{L}}$$

$$I = I_0 e^{-Rt/L} = \frac{E}{R}e^{-Rt/L}$$

$$V_R = IR = Ee^{-Rt/L}$$

$$V_L = L\frac{dI}{dt} = -\frac{E}{L}e^{-Rt/L}$$

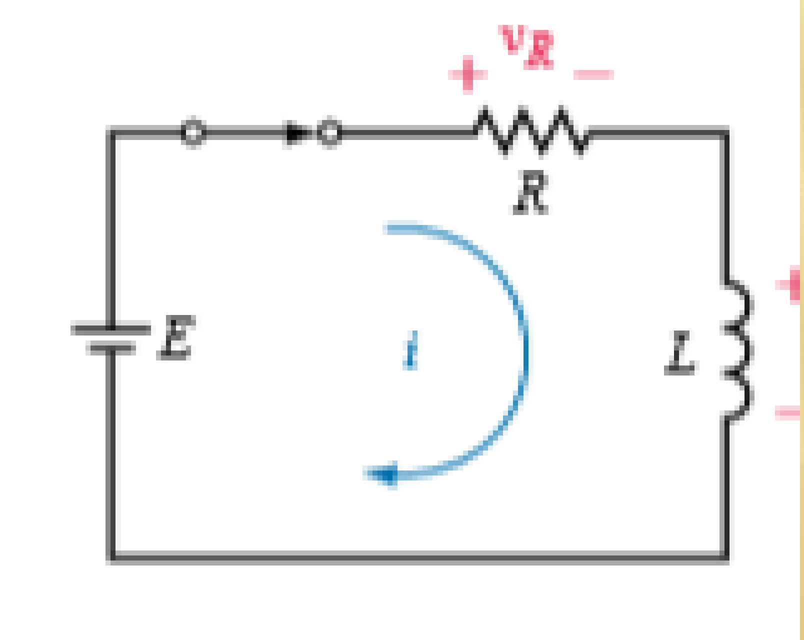
TIME CONSTANT & DURATION OF A TRANSIENT

- The time constant T=L/R and it determines the rate at which inductor energize or de-energize.
- The transient time depends on the exponential function. 99.3% of the transient takes place in the first 5T seconds.

PROBLEM

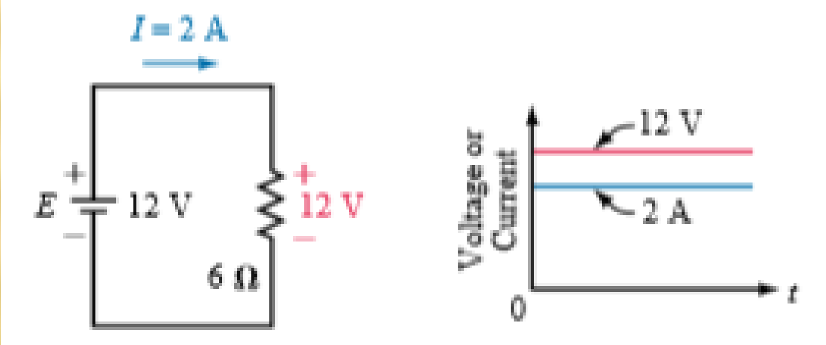
- If E=50 V, R=10Ω, and L=2H and it is in the energizing stage, find:

1. Expression for V_L and I_L
2. Plot the relations for V_L , I_L, V_R
3. What is the steady state current?

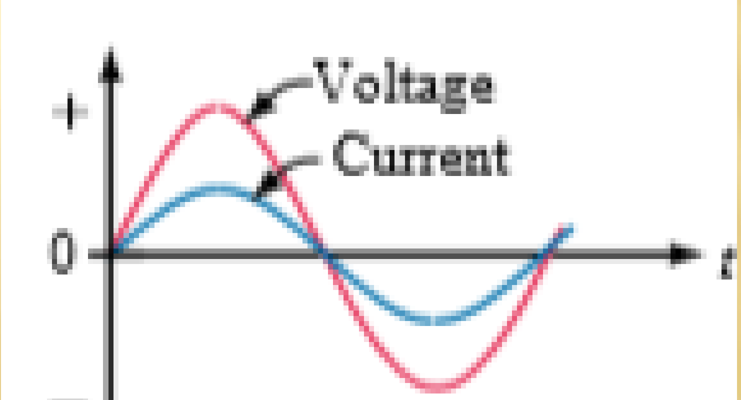


DC VERSUS AC SOURCES

- DC sources have constant magnitude and polarity while as AC sources changes both magnitude and polarity.
- The common ac sources used in our daily life take the form of sinusoidal change (the shape of a sine wave).



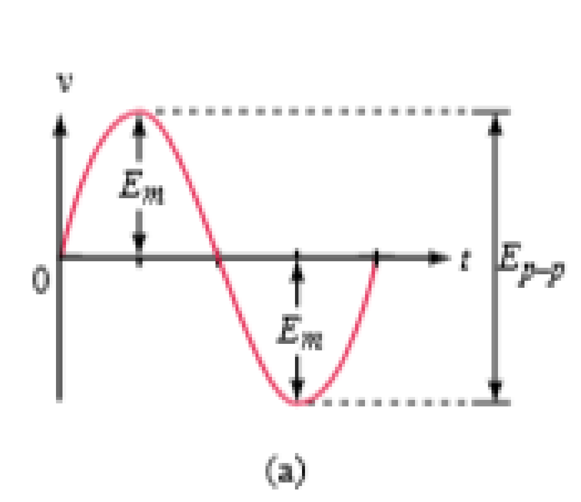
(a) (b) Voltage and current versus time for dc



AC current and voltage

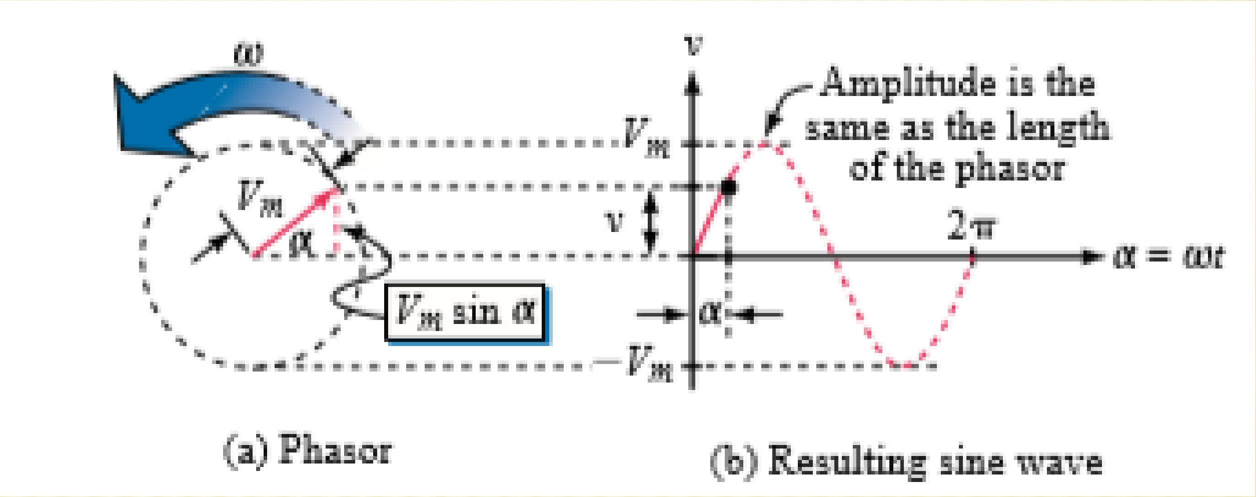
FREQUENCY, PERIOD, AMPLITUDE AND PEAK

- The frequency (*f*) of the waveform is the number of cycles per second and it measured in Hertz (Hz).
- The period (*T*) is the duration of one cycle or the inverse of the frequency $T=1/f$.
- The amplitude (V_m,I_m) is the distance from its average to its peak.
- Peak to Peak ($V_{p\text{-}p}$, $I_{p\text{-}p}$) is measured between minimum and maximum peaks.
- The angular velocity or angular frequency ($\omega=2\pi f=2\pi/T$).



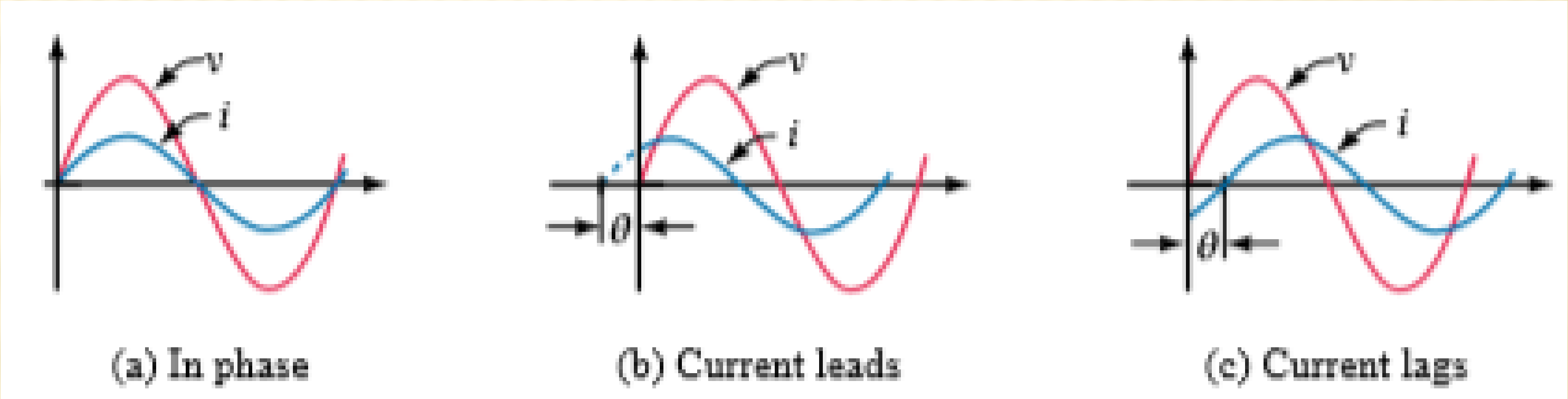
PHASORS

- The phasor is a rotating line whose projection on a vertical axis can be used to represent sinusoidal varying quantities.



PHASE DIFFERENCE

- The angular displacement between different waveforms of the same frequency.
- If the angular displacement is 0, they are in phase otherwise they are out of phase.



PROBLEM

- If V=20 sin(wt+30) and i=18 sin(wt-40), draw the waveforms and determine phase relationships.